Math 115

Spring 2017

Lecture 28

Solve:

$$\frac{2x}{x+2} - 2 = \frac{x-8}{x-2}$$

LCD $= (x+2)(x-2)$

E.V.: -2 & 2

$$(x+2)(x-2)\cdot\frac{2x}{(x+2)} - (x+2)(x-2)\cdot 2 = (x+2)(x-2)\cdot\frac{x-8}{x-2}$$

Conjugates

$$2x(x-2) - 2(x+2)(x-2) = (x+2)(x-8)$$

$$2x^2 - 4x - 2(x^2-4) = x^2 - 8x + 2x - 16$$

$$2x^2 - 4x - 2x^2 + 8 = x^2 - 6x - 16$$

$$x^2 - 6x - 16 + 4x - 8 = 0$$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$x = 6$ $\quad$ $x = -4$

$\{-4, 6\}$

A boat can travel 9 miles upstream in the same amount of time that goes 11 miles down stream. the current is 3 mph. Find speed of the boat in still water.

Cat.	r · t = d
upstream	$x-3 \cdot t = 9$
Downstream	$x+3 \cdot t = 11$

$t = \frac{9}{x-3}$, $t = \frac{11}{x+3}$

$\frac{9}{x-3} = \frac{11}{x+3}$

Cross-Multiply

$11(x-3) = 9(x+3)$

$11x - 33 = 9x + 27$

$2x = 60$

$x = 30$

30 mph

John can do certain job in 6 hrs, Jack can do the same job in 4 hrs. How long if they work together?

$\frac{1}{6} \cdot t + \frac{1}{4} \cdot t = 1$

LCD = 12

$2t + 3t = 12$

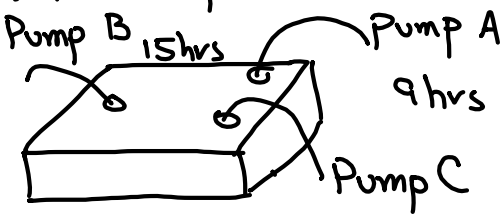
$5t = 12$

$t = \frac{12}{5}$

$t = 2.4$ | 2.4 hrs.

Pump A can fill up an empty pool in 9 hrs

Pump B ------ the same empty pool in 15 hrs. Pump C is used to empty the Pool. If all 3 pumps are working, it takes 5 hrs to fill it up. Find the rate for pump C.



$\frac{1}{9}\cdot 5 + \frac{1}{15}\cdot 5 + \frac{1}{x}\cdot 5 = 1$

$\frac{5}{9} + \frac{1}{3} + \frac{5}{x} = 1$

LCD = $9x$

$5x + 3x + 5\cdot 9 = 9x$

$\boxed{x = 45}$ 45 hrs.

Simplify: $\dfrac{\frac{x}{y} - \frac{9y}{x}}{\frac{1}{y} - \frac{3}{x}}$

LCD = xy

$$\frac{xy\cdot\frac{x}{y} - xy\cdot\frac{9y}{x}}{xy\cdot\frac{1}{y} - xy\cdot\frac{3}{x}}$$

$$= \frac{x^2 - 9y^2}{x - 3y} = \frac{(x+3y)(x-3y)}{x-3y}$$

$$= \boxed{x + 3y}$$

Simplify: $\dfrac{\frac{2}{y^2} - \frac{5}{xy} - \frac{3}{x^2}}{\frac{2}{y^2} + \frac{7}{xy} + \frac{3}{x^2}}$

LCD $= x^2y^2$

$$= \frac{2x^2 - 5xy - 3y^2}{2x^2 + 7xy + 3y^2}$$

$$= \frac{(2x+y)(x-3y)}{(2x+y)(x+3y)} = \boxed{\frac{x-3y}{x+3y}}$$

Quadratic Equation $ax^2 + bx + c = 0, \quad a \neq 0$

Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Solve $x^2 + 3x - 10 = 0$ by Q-formula.

$a = 1, \; b = 3, \; c = -10$

$b^2 - 4ac = (3)^2 - 4(1)(-10)$

$= 9 + 40 = 49$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{49}}{2(1)} = \frac{-3 \pm 7}{2}$$

$x = \frac{-3+7}{2} = \frac{4}{2} = 2$

$x = \frac{-3-7}{2} = \frac{-10}{2} = -5$

$\{-5, 2\}$

Solve $2x^2 - 3x - 5 = 0$ by Q-formula.

$ax^2 + bx + c = 0$

$a=2, \quad b=-3 \quad c=-5$

$b^2 - 4ac = (-3)^2 - 4(2)(-5) = 9 + 40 = \boxed{49}$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-3) \pm \sqrt{49}}{2(2)} = \frac{3 \pm 7}{4}$$

$x = \frac{3+7}{4} = \frac{5}{2}$

$x = \frac{3-7}{4} = -1$

$\left\{-1, \frac{5}{2}\right\}$

Solve by Q-formula

$(3x-1)(2x+3) = 35$

Foil & Simplify

To use Q-formula, Make sure Problem is in $ax^2 + bx + c = 0$.

$6x^2 + 9x - 2x - 3 - 35 = 0$

$6x^2 + 7x - 38 = 0$

$ax^2 + bx + c = 0$

$a=6, \quad b=7, \quad c=-38$

$b^2 - 4ac = (7)^2 - 4(6)(-38) = 961$

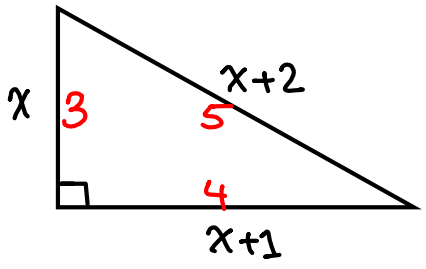
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-7 \pm \sqrt{961}}{2(6)} = \frac{-7 \pm 31}{12}$$

$x = \frac{-7+31}{12} = \frac{24}{12} = \boxed{2}$

$x = \frac{-7-31}{12} = \frac{-38}{12} = \boxed{\frac{-19}{6}}$

$\left\{2, \frac{-19}{6}\right\}$

Three sides of a right triangle are 3 Cons. intesers. find all three.



Using Pythagorean thm

$x^2 + (x+1)^2 = (x+2)^2$

$x^2 + (x+1)(x+1) = (x+2)(x+2)$

$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$

$x^2 - 2x - 3 = 0$

$a=1$ $b=-2$ $c=-3$

$b^2 - 4ac = (-2)^2 - 4(1)(-3)$

$= 16$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

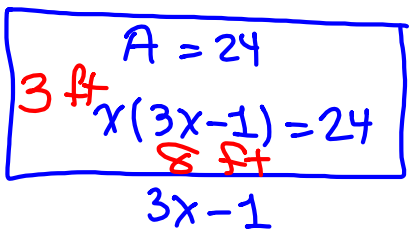
$x = \frac{-(-2) \pm \sqrt{16}}{2(1)}$

$= \frac{2 \pm 4}{2}$

$x = \frac{2+4}{2} = 3$ $x = \frac{2-4}{2} = -1$ ~~$= -1$~~

Area of a rectangle is 24 ft^2. The length is 1 ft shorter than 3 times its width.

1) Draw & label



2) find the eqn $3x^2 - x - 24 = 0$

3) use Q-formula to solve $a=3$, $b=-1$, $c=-24$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{289}}{6} = \frac{1 \pm 17}{6}$

$b^2 - 4ac = (-1)^2 - 4(3)(-24)$

$= 289$

$x = 3$

The sum of the reciprocal of two cons. even integers is $\frac{5}{12}$. Find all such integers.

Two cons. even integers : x & $x+2$

Reciprocals are : $\frac{1}{x}$, $\frac{1}{x+2}$

$$\frac{1}{x} + \frac{1}{x+2} = \frac{5}{12} \qquad \text{LCD} = 12x(x+2)$$

$$12(x+2) + 12x = 5x(x+2)$$

Simplify, and write in $ax^2+bx+c=0$ form.

$$5x^2+10x-12x-24-12x=0$$

$$5x^2-14x-24=0$$

$a=5$, $b=-14$, $c=-24$

$$b^2-4ac = (-14)^2-4(5)(-24) = \boxed{676}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-14) \pm \sqrt{676}}{2(5)}$$

$$= \frac{14 \pm 26}{10}$$

4 & 6

$$x = \frac{14+26}{10} \qquad x = \frac{14-26}{10}$$

$\boxed{x=4}$ $\quad$ ~~$x=-1.2$~~ Not an integer

Find eqn of a line that contains (3,-2) and (1,4).

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2-4}{3-1} = \frac{-6}{2} = \boxed{-3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -3(x-1)$$

$$y - 4 = -3x + 3 \Rightarrow \boxed{y = -3x + 7}$$

Solve

$$\begin{cases} 3x - 2y = 7 & \times 3 \\ 2x + 3y = -4 & \times 2 \end{cases} \Rightarrow \begin{cases} 9x - 6y = 21 \\ 4x + 6y = -8 \end{cases}$$

$$13x = 13$$

$$\boxed{x = 1}$$

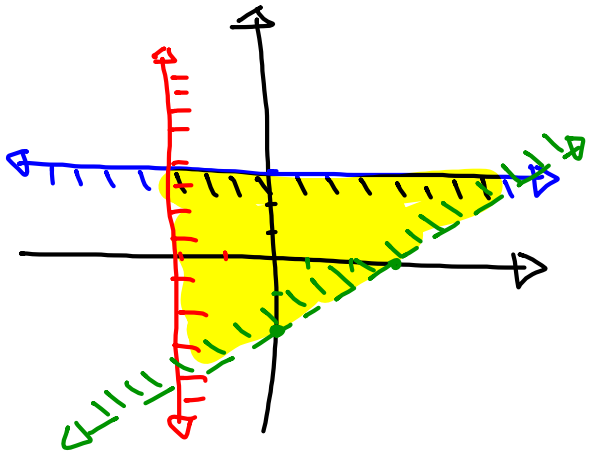
$$2(1) + 3y = -4$$

$$2 + 3y = -4 \rightarrow 3y = -6 \rightarrow \boxed{y = -2}$$

$$(1, -2)$$

Shade the Soln.

$$\begin{cases} y \le 3 \\ x \ge -2 \\ y > \frac{2}{3}x - 2 \end{cases}$$



Simplify

$$\left(\frac{-5x^{-4}}{y^{-6}}\right)^{-2} = \left(\frac{-5y^{6}}{x^{4}}\right)^{-2} = \left(\frac{x^{4}}{-5y^{6}}\right)^{2}$$

$$= \frac{x^{8}}{25y^{12}}$$

Factor

① $x^4 - 3x^3 - 4x^2 + 12x$

$= x(x^3 - 3x^2 - 4x + 12)$

$= x[x^2(x-3) - 4(x-3)] = x(x-3)(x^2-4)$

$= \boxed{x(x-3)(x+2)(x-2)}$

② $2x^5 - 250x^2$

$= 2x^2(x^3 - 125)$

$= 2x^2(x^3 - 5^3) = \boxed{2x^2(x-5)(x^2+5x+25)}$

Solve, graph the Soln. Give Ans in interval notation.

$-5 < 2x + 1 \leq 9$

$-5-1 < 2x \leq 9-1$

$-6 < 2x \leq 8$

$-3 < x \leq 4$

S.B.N. $\{x \mid -3 < x \leq 4\}$

I.N. $(-3, 4]$

Simplify

$(2x-5)^2 + 20x - 25$

$= (2x-5)(2x-5) + 20x - 25$

$=$

$= 4x^2$

Divide

$\frac{x^3 + 5x^2 - 7x + 1}{x-2}$

$x \boxed{x^2} = x^3$

$x \boxed{-7x} = -7x^2$

$x \boxed{-7} = -7x$

$$\begin{array}{r} x^2 + 7x + 7 \\ x-2 \overline{)\, x^3 + 5x^2 - 7x + 1} \\ -(x^3 - 2x^2 \quad) \\ \hline 7x^2 - 7x + 1 \\ -(7x^2 - 14x \quad) \\ \hline 7x + 1 \\ -(7x - 14) \\ \hline 15 \end{array}$$

$$x^2 + 7x + 7 + \frac{15}{x-2}$$